Milestone M5 – Technical Brief Draft

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From: Brandon Reid, Paige Stanfill, Alyssa Cwidak, & Graden Young
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RE: Final technical brief for the SCGL project
Date: 4/25/18

First-Order Systems, Inc. (FOS) is a technology company that specializes in measuring physical phenomena, like temperature. Currently, they are testing five new thermocouple designs. The test conditions are relevant to their client's (Swiss Chocolatiers of Greater Lafayette) needs. They need precise temperature control in their manufacturing processes. FOS needs us to help characterize the performance of the thermocouples and begin the sales literature. We have been tasked with creating an algorithm that analyzes the provided time histories for 20 samples of 10 heating and 10 cooling data sets for each of the 5 FOS designs to find a time parameter for each design. The algorithm should model yH, yL, ts, tau where yH and yL are the asymptotic high and low temperatures, ts is the time the temperature begins to change and, tau is the time it takes for the temperature to reach 63.2% of its final temperature starting from ts. A successful algorithm for determining the parameters that results in low SSEmod for individual time histories and/or low tau standard deviation for a given FOS design. We must provide an error analysis of the accuracy of our algorithm. We must thoroughly assess the performance of the designs and give a recommendation to FOS on what they can tell consumers about their designs. They want to give their customers a guarantee that their thermocouples are consistent in performance which means a consistent time constant.

The algorithm accepts the time and temperature from the time histories as inputs and calculates the parameters (yH, yL, tau, and ts) to model the data as best as possible. The tau value is varied to maximize the r-squared value and thus model the data with the least amount error.

During the development of our algorithm, several improvements were made. First off, by deciding to increase the number of starting points in the loop for finding the final temperature the SSE_mod decreased as a result of the parameter fitting the data fit and the noise having less effect. Secondly, using a starting point to test tau, the program efficiency increased over tenfold as the program no longer had to run a loop thousands of times per data set. A third improvement; we decided to switch the method of finding ts to finding the first point where the temperature exceeds the average starting temperature. Ts became more accurate and more precise and in turn made SSE_mod lower because the tau calculation uses ts as a parameter and an inaccurate ts led to an inaccurate tau which created a model that doesn't follow the data.

The first step in the parameter identification algorithm is to determine if the data is "Heating" or "Cooling" by comparing the first temperature data value to the last temperature data value. The starting temperature is then calculated by averaging all the data points that fall within the first three-quarters of a second. To then find the final temperature, loop values and initialized including a vector comprised of the last seventy-five temperature data values, the index of the beginning of the vector and the standard deviation of the data in the vector. A loop repeats the following, the final temperature is equal to the average of the values in the temperature vector, then adds a new value to the vector, calculates the standard deviation and compares it to the previous standard deviation. If the standard deviation has changed by more than one percent, the loop terminates, else the loop repeats until this condition is met.

The next step is to determine the ts value. Ts is calculated by first creating a vector of all the temperature values above the starting temperature if "Cooling" or below the starting temperature if "Heating". Ts is then the time value that corresponds to the last data point in this new vector. If "Heating", yL is set equal to the starting temperature and yH equal to the final temperature. "Cooling" is the opposite.

With yL, yH, and ts now determined, these values will now be used to determine tau. This is through a binary search. The first step is to determine the starting values for tau. The lower bound of tau is calculated as the time to reach 57.5 percent of the temperature change. The upper bound is equal to the last time value minus ts, and the midpoint is halfway between the two. The SST value of the data set is calculated, as r-squared will be used later. Models are created for each of the three tau values using the equations found in the appendix, "Heating" (equation 1) and "Cooling" (equation 2). Each model's SSE and r-squared values are calculated. Next, the two tau values with the r-squared values closest to one (1) become the new bounds. The midpoint is updated, and this process repeats until all three converge into a single tau value. Once complete, yL, yH, ts, and tau are passed back to the executive function.

After the parameter identification process, our results were very consistent over the course of the one-hundred time histories. As shown in Table 1, for each of the five different types of thermocouples, the SSE_mod, or error squared per data point was between 0.3 and 0.4 [°F^2]. This indicated that the models created from the identified parameters consistently fit the data well. In the appendix, figures (1) and (2) show examples of the model overlapping the data for both the "Heating" and "Cooling" conditions. Also shown in the table below, all the standard deviations for tau are under 0.04. This also indicates a consistency in determining tau from data set to data set.

From our regression analysis of price, in dollars, as a function of tau, in seconds, we determined that an exponential relationship (shown in figure 3) is present following the following equation:

$$Price[\$] = 21.59 * 10^{-0.90\tau}$$

This regression model had a SSE value of 91.260 [$\$^2$], a SST value of 3547.800 [$\$^2$], and a r-squared value of 0.974. This means that the model explains 97.4% of the variability in the price and thus high quality.

The error in this data can be processed and interrupted many ways. Overall the quality of the experiments was good. The time during each experiment was the same. This made comparing experiment to experiment very easy. Also, our parameter identification algorithm is of good quality as it consistently produced parameters that accurately fit the data. This is shown in the average SSE_mod values all being between 0.3 and 0.4 [°F^2]. From our analysis, we believe FOS is accurate about their product's performance, price, and consistency. All time histories followed a first order differential equation, indicating proper performance. The pricing nearly followed a nice exponential curve, and with the tau standard deviation values being less than 0.04 for all five of the thermocouple, this shows that the manufacturing if the thermocouples is very consistent.

Appendix:

Equation 1:

$$y(t) = \begin{cases} y_L & ; \ t < t_s \\ y_L + (y_H - y_L)[1 - \exp(\frac{t - t_s}{\tau})] & ; \ t \geq t_s \end{cases}$$

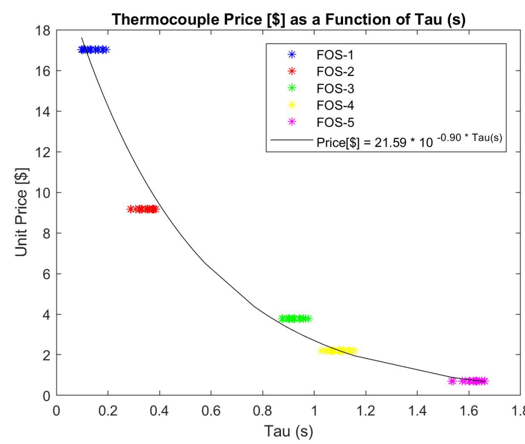
Equation 2:

$$y(t) = \begin{cases} y_H & ; \ t < t_s \\ y_L + (y_H - y_L)[\exp(\frac{t - t_s}{\tau})] & ; \ t \geq t_s \end{cases}$$

Table 1:

Model Number	M4 Algorithm		
	Tau Characteristics		Mean SSE_{mod} [°F²]
	Mean [sec]	Standard Deviation [sec]	
FOS-1	0.14	0.029	0.33
FOS-2	0.34	0.029	0.34
FOS-3	0.92	0.030	0.35
FOS-4	1.09	0.035	0.35
FOS-5	1.62	0.038	0.38

Figure 3:



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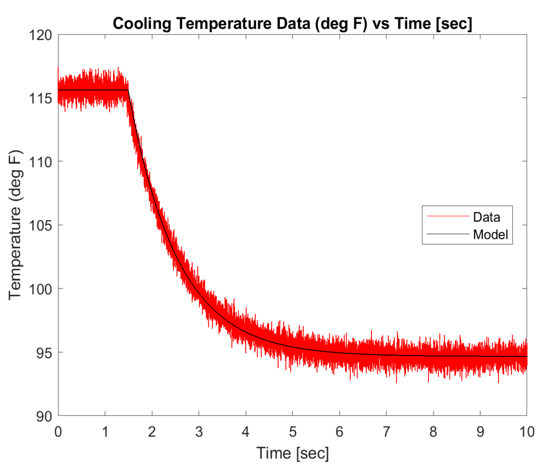

Figure 2: